5.5 - Performing Function Operations

Warmup

Find the domain expressed in interval notation.

1. $f(x) = \dfrac{x+1}{x+4}$

$(-\infty, -4)$
or $(-4, +\infty)$

2. $f(x) = \sqrt{x+1}$

$[-1, +\infty)$

3. $f(x) = \dfrac{1}{\sqrt{x+1}}$

$(-1, +\infty)$

4. $f(x) = \sqrt{\dfrac{1}{x}}$

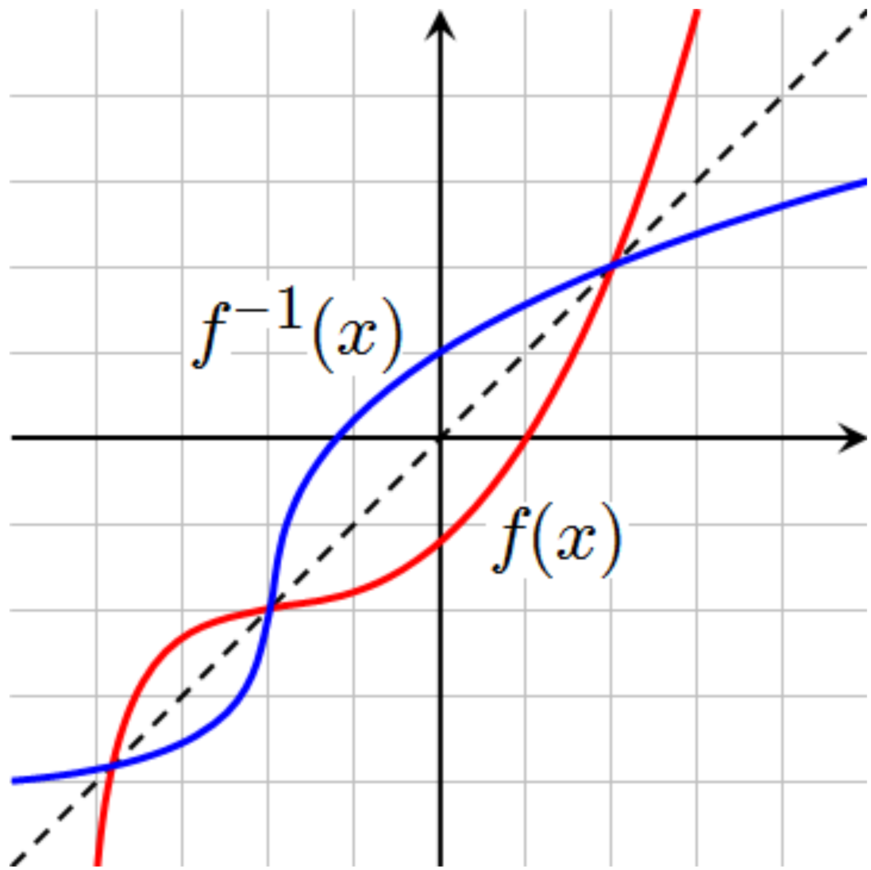
$(0, +\infty)$

5. $f(x) = 4$

$(-\infty, +\infty)$

5.6 - Inverse of a Function

Functions that undo each other are called **inverse functions**.



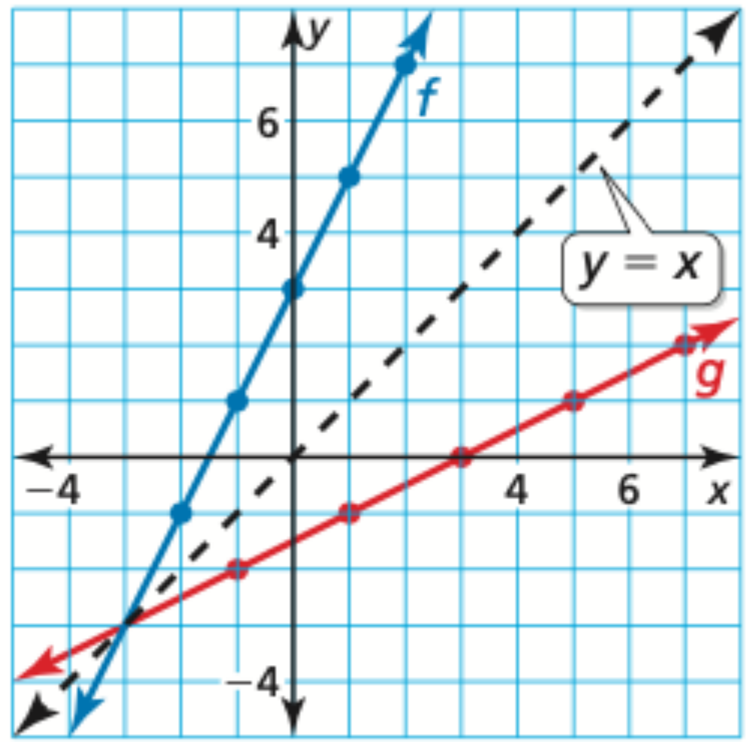
5.6 - Inverse of a Function

Functions that undo each other are called **inverse functions**.

Original $f(x) = 2x + 3$

Inverse

$$g(x) = f^{-1}(x) = \frac{x-3}{2}$$



5.6 - Inverse of a Function

Inverse Functions

$g(x) = 2x - 2$

Find $g^{-1}(x)$

Swap x and y

$$x = 2y - 2$$

$$x + 2 = 2y$$

$$\frac{x+2}{2} = y$$

$$g^{-1}(x) = \frac{x+2}{2}$$

"g inverse of x"

5.6 - Inverse of a Function

Inverse Functions

$g(x) = 2x - 2$

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$$g^{-1}(x) = \frac{x+2}{2}$$

"g inverse of x"

Practice - Find inverse

1. $f(x) = 3x + 2$

   $f^{-1}(x) = \dfrac{x-2}{3}$

2. $f(x) = 2x^2 - 5$

   $f^{-1}(x) = \pm\dfrac{\sqrt{2x+10}}{2}$

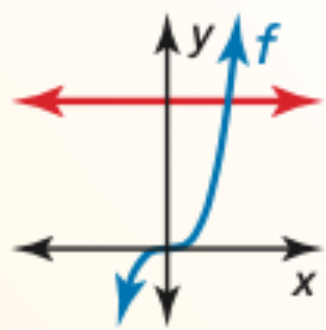
3. $f(x) = \dfrac{12}{2x-4}$

   $f^{-1}(x) = \dfrac{6}{x} + 2$

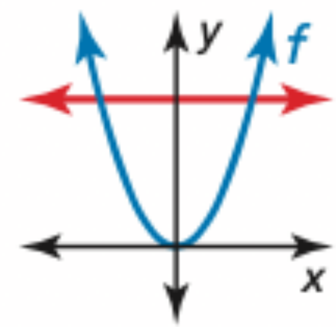
5.6 - Inverse of a Function

Horizontal Line Test

The inverse of a function f is also a function if and only if no horizontal line intersects the graph of f more than once.



$f(x) = x^3$



$f(x) = x^2$

5.6 - Inverse of a Function

Inverse Functions

Find $f(f^{-1}(x))$

1. $f(x) = 3x + 2$

   $f^{-1}(x) = \dfrac{x-2}{3}$

   $f(f^{-1}(x)) = x$

2. $f(x) = 2x^2 - 5$

   $f^{-1}(x) = \pm\dfrac{\sqrt{2x+10}}{2}$

   $f(f^{-1}(x)) = x$

3. $f(x) = \dfrac{12}{2x-4}$

   $f^{-1}(x) = \dfrac{6}{x} + 2$

   $f(f^{-1}(x)) = x$

What about $f^{-1}(f(x))$?

5.6 - Inverse of a Function

Inverse Functions

$f(x) = \sqrt{2x - 3}$

Find $f^{-1}(3)$

$$x = \sqrt{2y - 3}$$

$$x^2 = 2y - 3$$

$$x^2 + 3 = 2y$$

$$f^{-1}(x) = \frac{x^2 + 3}{2}$$

$$f^{-1}(3) = \frac{3^2 + 3}{2}$$

$$f^{-1}(3) = \frac{12}{2}$$

$$f^{-1}(3) = 6$$

Faster Way $3 = \sqrt{2y - 3}$

5.6 - Inverse of a Function

Inverse Functions

Practice

Find $f^{-1}(-1)$

1. $f(x) = \dfrac{1}{2x+1}$

$f^{-1}(-1) = -1$

2. $f(x) = \dfrac{1}{\sqrt{x+1}}$

No solution

5.6 - Inverse of a Function

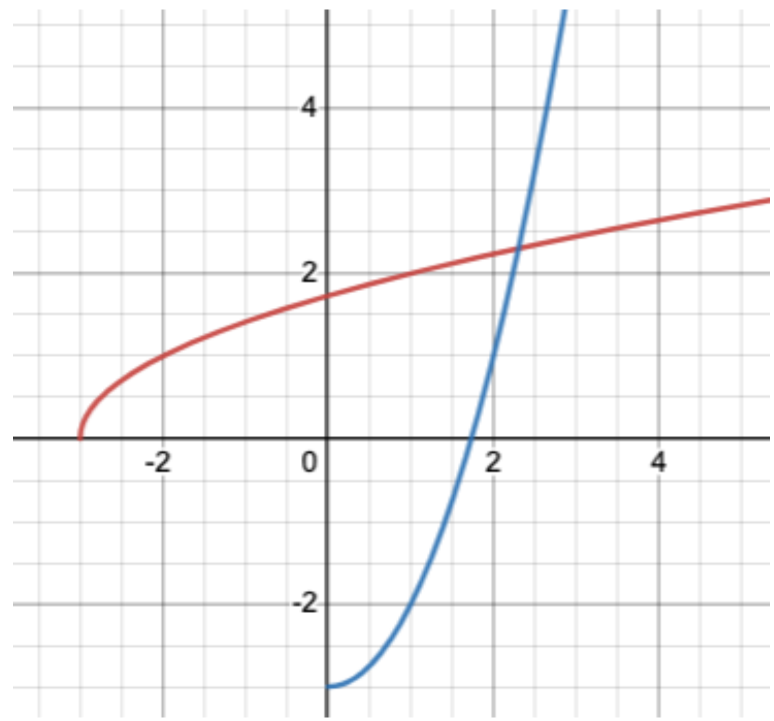
Inverse Functions

Find the domain and range of f^{-1}

$f(x) = \sqrt{x+3}$

$f^{-1}(x) = x^2 - 3$

	$f(x)$	$f^{-1}(x)$
D:	$x \geq -3$	$x \geq 0$
R:	$y \geq 0$	$y \geq -3$



5.6 - Inverse of a Function

Inverse Functions

Find the domain and range of f^{-1}

1. $f(x) = \sqrt{3x+2}$

$$f^{-1}(x) = \frac{x^2-2}{3}$$

$D:\ x \geq 0$

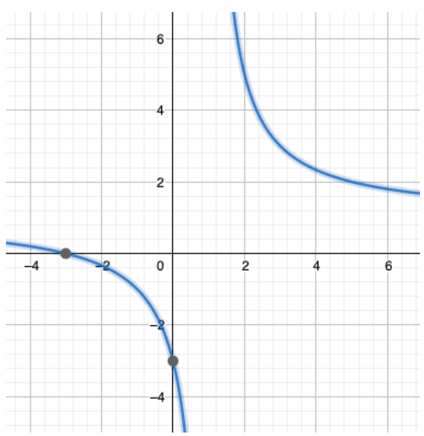
$R:\ y \geq -\frac{2}{3}$

2. $f(x) = \frac{x+3}{x-1}$

$$f^{-1}(x) = \frac{x+3}{x-1}$$

$D:\ x \neq 1$

$R:\ y \neq 1$



Because: (using synthetic division)

$$f(x) = \frac{x+3}{x-1} = 1 + \frac{4}{x-1}$$

5.6 - Inverse of a Function

Inverse Functions

Find the domain and range of f^{-1} where $f(x) = g(k(x))$

$k(x) = \dfrac{1}{x+2}$ $\qquad$ $g(x) = \sqrt{x^{-1} + 3}$

$D\ k(x): \ x \neq -2$

$g(k(x)) = \sqrt{x+5}$

$D\ g(k(x)): \ x \geq -5 \ and \ x \neq -2$

$R\ g(k(x)): \ y \geq 0 \ and \ y \neq \sqrt{3}$

$f^{-1}(x)$

$x = \sqrt{y+5}$

$x^2 = y + 5$

$x^2 - 5 = y$

$f^{-1}(x) = x^2 - 5$

$D\ f^{-1}(x): \ x \geq 0 \ and \ x \neq \sqrt{3}$

$R\ f^{-1}(x): \ y \geq -5 \ and \ y \neq -2$

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